NAG Toolbox for MATLAB

e02df

1 Purpose

e02df calculates values of a bicubic spline from its B-spline representation. The spline is evaluated at all points on a rectangular grid.

2 Syntax

```
[ff, ifail] = e02df(x, y, lamda, mu, c, 'mx', mx, 'my', my, 'px', px, 'py', py)
```

3 Description

e02df calculates values of the bicubic spline $s(x,y)$ on a rectangular grid of points in the x-y plane, from its augmented knot sets $\{\lambda\}$ and $\{\mu\}$ and from the coefficients c_{ij}, for $i = 1,2,\ldots,\mathbf{px}-4$ and $j = 1,2,\ldots,\mathbf{py}-4$, in its B-spline representation

$$s(x,y) = \sum_{ij} c_{ij} M_i(x) N_j(y).$$

Here $M_i(x)$ and $N_j(y)$ denote normalized cubic B-splines, the former defined on the knots λ_i to λ_{i+4} and the latter on the knots μ_j to μ_{j+4}.

The points in the grid are defined by co-ordinates x_q, for $q = 1,2,\ldots,m_x$, along the x axis, and co-ordinates y_r, for $r = 1,2,\ldots,m_y$ along the y axis.

This function may be used to calculate values of a bicubic spline given in the form produced by e01da, e02da, e02dc and e02dd. It is derived from the function B2VRE in Anthony *et al.* 1982.

4 References

Anthony G T, Cox M G and Hayes J G 1982 *DASL – Data Approximation Subroutine Library* National Physical Laboratory

Cox M G 1978 The numerical evaluation of a spline from its B-spline representation *J. Inst. Math. Appl.* **21** 135–143

5 Parameters

5.1 Compulsory Input Parameters

1: **x(mx) – double array**

2: **y(my) – double array**

x and **y** must contain x_q, for $q = 1,2,\ldots,m_x$, and y_r, for $r = 1,2,\ldots,m_y$, respectively. These are the x and y co-ordinates that define the rectangular grid of points at which values of the spline are required.

Constraint: **x** and **y** must satisfy

$$\mathbf{lamda}(4) \le \mathbf{x}(q) < \mathbf{x}(q+1) \le \mathbf{lamda}(\mathbf{px}-3), \qquad q = 1,2,\ldots,m_x-1$$

and

$$\mathbf{mu}(4) \le \mathbf{y}(r) < \mathbf{y}(r+1) \le \mathbf{mu}(\mathbf{py}-3), \qquad r = 1,2,\ldots,m_y-1.$$

The spline representation is not valid outside these intervals.

3: **lamda**(**px**) – **double array**

4: **mu**(**py**) – **double array**

lamda and **mu** must contain the complete sets of knots $\{\lambda\}$ and $\{\mu\}$ associated with the x and y variables respectively.

Constraint: the knots in each set must be in nondecreasing order, with **lamda**(**px** $-$ 3) $>$ **lamda**(4) and **mu**(**py** $-$ 3) $>$ **mu**(4).

5: **c**((**px** $-$ **4**) $\times$ (**py** $-$ **4**)) – **double array**

c((**py** $-$ 4) $\times$ $(i-1)+j$) must contain the coefficient c_{ij} described in Section 3, for $i = 1, 2, \ldots,$ **px** $-$ 4 and $j = 1, 2, \ldots,$ **py** $-$ 4.

5.2 Optional Input Parameters

1: **mx** – **int32 scalar**

2: **my** – **int32 scalar**

Default: The dimension of the arrays **x**, **y**. (An error is raised if these dimensions are not equal.)

mx and **my** must specify m_x and m_y respectively, the number of points along the x and y axis that define the rectangular grid.

Constraint: **mx** ≥ 1 and **my** ≥ 1.

3: **px** – **int32 scalar**

4: **py** – **int32 scalar**

Default: For **px**, the dimension of the array **lamda**. For **py**, the dimension of the array **mu**.

px and **py** must specify the total number of knots associated with the variables x and y respectively. They are such that **px** $-$ 8 and **py** $-$ 8 are the corresponding numbers of interior knots.

Constraint: **px** ≥ 8 and **py** ≥ 8.

5.3 Input Parameters Omitted from the MATLAB Interface

wrk, lwrk, iwrk, liwrk

5.4 Output Parameters

1: **ff**(**mx** $\times$ **my**) – **double array**

ff(**my** $\times$ $(q-1)+r$) contains the value of the spline at the point (x_q, y_r), for $q = 1, 2, \ldots, m_x$ and $r = 1, 2, \ldots, m_y$.

2: **ifail** – **int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **mx** < 1,
or **my** < 1,
or **py** < 8,
or **px** < 8.

ifail $= 2$

On entry, **lwrk** is too small,
or **liwrk** is too small.

ifail $= 3$

On entry, the knots in array **lamda**, or those in array **mu**, are not in nondecreasing order, or $\mathbf{lamda}(\mathbf{px}-3) \leq \mathbf{lamda}(4)$, or $\mathbf{mu}(\mathbf{py}-3) \leq \mathbf{mu}(4)$.

ifail $= 4$

On entry, the restriction $\mathbf{lamda}(4) \leq \mathbf{x}(1) < \cdots < \mathbf{x}(\mathbf{mx}) \leq \mathbf{lamda}(\mathbf{px}-3)$, or the restriction $\mathbf{mu}(4) \leq \mathbf{y}(1) < \cdots < \mathbf{y}(\mathbf{my}) \leq \mathbf{mu}(\mathbf{py}-3)$, is violated.

7 Accuracy

The method used to evaluate the B-splines is numerically stable, in the sense that each computed value of $s(x_r, y_r)$ can be regarded as the value that would have been obtained in exact arithmetic from slightly perturbed B-spline coefficients. See Cox 1978 for details.

8 Further Comments

Computation time is approximately proportional to $m_x m_y + 4(m_x + m_y)$.

9 Example

```
x = [1;
     1.1;
     1.3;
     1.4;
     1.5;
     1.7;
     2];
y = [0;
     0.2;
     0.4;
     0.6;
     0.8;
     1];
lamda = [1;
     1;
     1;
     1;
     1.3;
     1.5;
     1.6;
     2;
     2;
     2;
     2];
mu = [0;
     0;
     0;
     0;
     0.4;
     0.7;
     1;
     1;
     1;
     1];
c = [1;
     1.1333;
```

```
     1.3667;
     1.7;
     1.9;
     2;
     1.2;
     1.3333;
     1.5667;
     1.9;
     2.1;
     2.2;
     1.5833;
     1.7167;
     1.95;
     2.2833;
     2.4833;
     2.5833;
     2.1433;
     2.2767;
     2.51;
     2.8433;
     3.0433;
     3.1433;
     2.8667;
     3;
     3.2333;
     3.5667;
     3.7667;
     3.8667;
     3.4667;
     3.6;
     3.8333;
     4.1667;
     4.3667;
     4.4667;
     4;
     4.1333;
     4.3667;
     4.7;
     4.9;
     5];
[ff, ifail] = e02df(x, y, lamda, mu, c)
```

```
ff =
    1.0000
    1.2000
    1.4000
    1.6000
    1.8000
    2.0000
    1.2100
    1.4100
    1.6100
    1.8100
    2.0100
    2.2100
    1.6900
    1.8900
    2.0900
    2.2900
    2.4900
    2.6900
    1.9600
    2.1600
    2.3600
    2.5600
    2.7600
    2.9600
    2.2500
    2.4500
    2.6500
```

```
    2.8500
    3.0500
    3.2500
    2.8900
    3.0900
    3.2900
    3.4900
    3.6900
    3.8900
    4.0000
    4.2000
    4.4000
    4.6000
    4.8000
    5.0000
ifail =
           0
```